

# INTEGRATED MICROWAVE FIELD SIMULATION USING THREE DIMENSIONAL FINITE ELEMENTS

Z. J. Cendes, J. F. Lee, D. N. Shenton,  
B. Shyamkumar, E. Smetak and D. K. Sun  
ANSOFT Corp.  
Four Station Square  
Pittsburgh, PA 15219

## ABSTRACT

A method for microwave field simulation based on three-dimensional finite elements is described. The method employs solid modeling for geometry generation, Delaunay tessellation for mesh generation,  $\chi_1(\text{curl})$  tangential vector finite elements for field solution, and transfinite elements for port representation. Applications to conventional and MMIC devices are described.

## INTRODUCTION

A number of features of the finite element method make it attractive for microwave field simulation. It is flexible and can model devices containing complicated shapes and a variety of different materials. It is efficient since the sparse structure of finite element coefficient matrix provides great computational savings and the matrix elements themselves can be precomputed. However, some other features of the finite element method have until now prevented its use as a common three-dimensional microwave design tool. Data entry and mesh generation with the finite element method has been difficult, conventional finite element methods have produced spurious modes, and methods to couple the input and output ports to the finite element method have been inefficient.

This paper presents several new procedures for microwave field simulation by the finite element

method that overcomes these limitations. First, we employ solid modeling to simplify the entry of the device geometry. Second, we employ Delaunay tessellation to generate three-dimensional finite element meshes without any user input. Third, we employ  $\chi_1(\text{curl})$  tangential vector finite element basis functions that eliminate the problem of spurious modes found with other methods. And fourth, we introduce the use of transfinite element method in three dimensions to couple the field solution at the input and output ports.

## SOLID MODELING

Electromagnetic field simulation begins with the problem of representing the shape of device under study in the computer. The approach used in this paper is to introduce the techniques of constructive solid geometry developed in the computer graphics area to define the shapes of microwave devices. Constructive solid geometry is a procedure for modeling solids in a computer by employing the Euler invariance properties of three-dimensional objects [1]. A key aspect of these Euler invariance relationships is that complicated

structures may be built up out of simple solids by computing the Boolean unions and intersections of these solids. For example, the coax to waveguide transition in Figure 1 was entered in the computer by using this process.

## DELAUNAY MESH GENERATION

The application of the Delaunay tessellation to finite element mesh generation in two dimensions is described in reference [2]. Delaunay tessellation is based on the concept that an optimal triangular mesh is one in which no triangle circumcircle contains any other triangle vertex in its interior. This method can be extended to three dimensions by employing a similar relationship between the tetrahedrons of a three-dimensional mesh and the corresponding circumspheres [3]. This is the approach used in this paper. Figure 1 also shows the Dealunay mesh for the coax to waveguide converter.

## TANGENTIAL VECTOR FINITE ELEMENTS

As described in reference [4], new types of basis functions called tangential vector finite elements eliminate the problem of spurious modes that have made three-dimensional finite element field solution unreliable in the past. The work reported here uses the  $\mathcal{X}_1(\text{curl})$  elements. The  $\mathcal{X}_1(\text{curl})$  elements can be described mathematically as

$$\mathcal{X}_1(\text{curl}) = \{ \mathcal{A} \mid \mathcal{A} \in [\mathcal{L}_2(\Omega)]^3, \nabla \times \mathcal{A} \in [\mathcal{P}_1(\Omega)]^3 \} \quad (1)$$

where  $\mathcal{L}_2(\Omega)$  is the set of square integrable functions, and  $\mathcal{P}_1(\Omega)$  is the set of piecewise polynomial functions complete to first order in the problem domain  $\Omega$ .

## TRANSFINITE ELEMENTS

A method for coupling the input and output fields at microwave ports is presented in reference [5] for two-dimensional planar circuits. Called the transfinite element method, the method employs what effectively one-dimensional analytical solutions in the port regions as basis functions within a variational framework to approximate the fields in the ports. This method is extended to three-dimensional problems in the present work. Since analytical solutions are not known in two dimensions in general, these field solutions are obtained by using two-dimensional finite element analysis as described in reference [4].

## APPLICATIONS

A wide number of three-dimensional microwave components have been analyzed by the procedure reported in this paper. Typical results are those shown in Figures 1 and 2. Figure 1 shows the analysis of a coax to waveguide converter. The solution time per frequency point for this problem using a 9530 by 9530 sparse matrix was about one half hour on an HP 835 computer. The product specification for the converter is 40 dB return loss; this result is well within the computed results shown in Table 1. Figure 2 shows the analysis of a microstrip filter. Again, the computed values of  $S_{11}$  agree closely with the measured values.

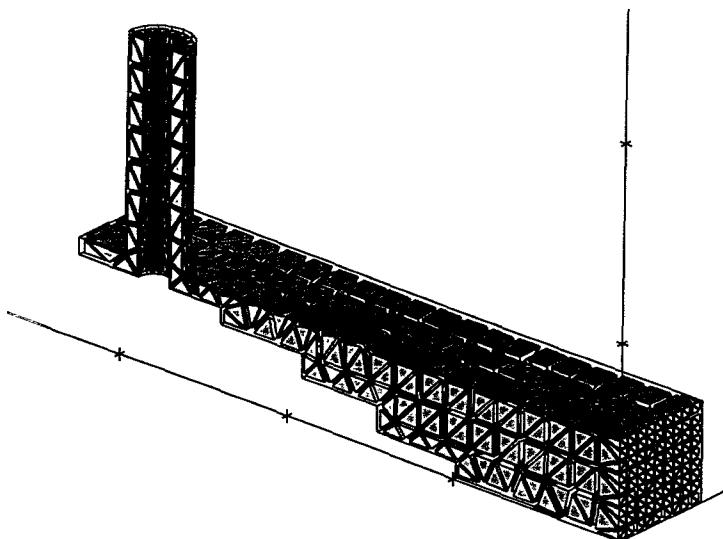
## References

- [1] M. E. Mortenson, "Geometric Modeling", *John Wiley*, New York 1985.
- [2] Z. J. Cendes, D. N. Shenton and H. Shahnasser, "Magnetic field computation using Delaunay triangulation and complementary finite element methods", *IEEE Trans. Magn.*, vol. MAG-19, pp. 2551-2554, 1983.
- [3] D. N. Shenton and Z. J. Cendes, "Three-dimensional finite element mesh generation

using Delaunay tessellation", *IEEE Trans. Magn.*, vol. MAG-21, pp. 2535-2538, 1985.

[4] J. F. Lee, "Finite element methods for modeling passive microwave devices", Ph. D. dissertation, *ECE Department, Carnegie Mellon University*, June 1989.

[5] Z. J. Cendes and J. F. Lee, "The transfinite element method for modeling MMIC devices", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-36, pp. 1639-1649, Dec. 1988.



```

-----  

freq = 9.00000e+03    phase = -8.839575e+01    db = 37.025  

mag = 1.408487e-02    phase = 4.802226e+01    db = 0.001  

mag = 9.999008e-01  

freq = 9.50000e+03    phase = 8.144082e+01    db = 51.094  

mag = 2.787907e-03    phase = -1.727506e+01    db = 0.000  

mag = 9.999961e-01  

freq = 1.00000e+04    phase = -1.228396e+01    db = 38.012  

mag = 1.257742e-02    phase = -8.031342e+01    db = 0.001  

mag = 9.999210e-01  

freq = 1.05000e+04    phase = -6.526523e+01    db = 37.669  

mag = 1.307895e-02    phase = 3.819505e+01    db = 0.001  

mag = 9.999145e-01  

freq = 1.10000e+04    phase = 8.437742e+01    db = 39.266  

mag = 1.088190e-02    phase = -2.226889e+01    db = 0.001  

-----
```

Figure 1: (a) Geometry of one half of a coax to waveguide transition showing the Delaunay mesh. (b) Computed S parameters at five different frequencies.

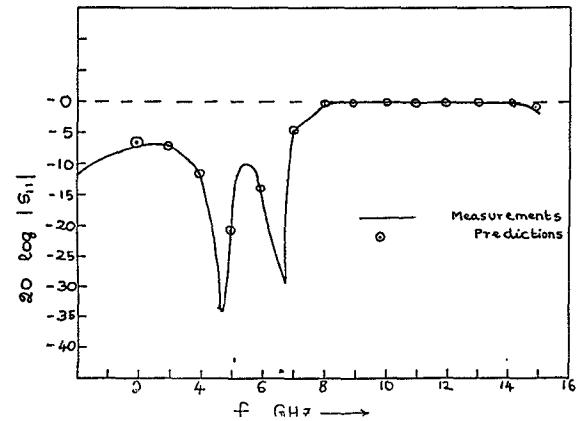
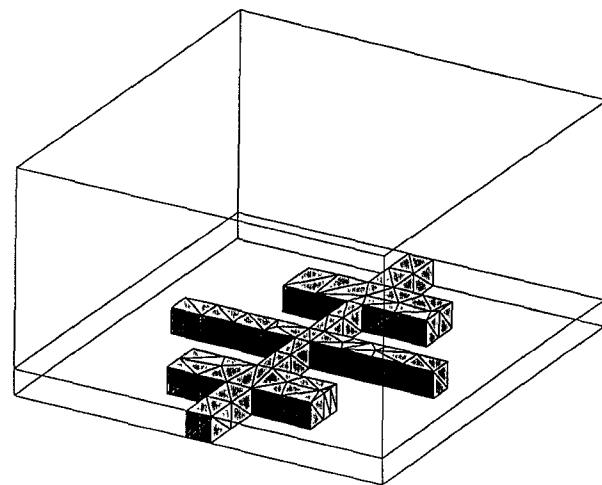


Figure 2: (a) Geometry of a microstrip filter.  
(b) Computed and measured return loss.